Practice 9

Topic **Check of the stability at the first approximation on Lyapunov's**

Let description of dynamic system we have in state-space of condition in the matrix form of the following kind:

$$\dot{X} = AX + BU, \qquad (1)$$

where *X [n*1]* is a vector of a state of Control Object (CO); *U[m*1]* is a control vector *(m≤n);*
*A [n*n], B[n*m]* are the matrixes of constant factors.

The characteristic equation of this system calls the name as follows:

$$\det\left(A - \lambda I\right) = 0 \;, \qquad (2)$$

where *I [n*n]* is an individual matrix of the appropriate dimension;

λ is an own numbers of the matrix A or roots of the characteristic equation of the following polynomial:

$$\lambda^n + a_1\lambda^{n-1} + \ldots a_{n-1}\lambda + a_n = 0.$$

We check of the dynamic system on stability, using the theorems Lyapunov's.

Algorithm

1. It is necessary to solve the characteristic equation of a kind (2):

$$\det\begin{vmatrix} (a_{11}-\lambda) & a_{12} & \ldots & a_{1n} \\ a_{21} & (a_{22}-\lambda) & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & (a_{nn}-\lambda) \end{vmatrix} = 0 \Rightarrow \lambda_i = \alpha_i \pm j\beta_i \quad \forall i = \overline{1,n}.$$

2. You should find of the own numbers of the matrix A or roots of the characteristic equation of the following polynomial:

$$\lambda^n + a_1\lambda^{n-1} + \ldots a_{n-1}\lambda + a_n = 0.$$

3. You should make a conclusion on stability of dynamic system of the found roots of the characteristic equation, using the theorems Lyapunov's.

Example. Let's description of dynamic system is given in state-space of condition in the matrix form of the following kind:

$$\dot{X} = AX + BU, \quad U(t) \equiv 0.$$

where the matrixes $A = \begin{bmatrix} -4 & 5 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Determine stability of dynamic system according to Lyapunov. Give of the geometrical interpretation.

Algorithm and solution

1. Write the characteristic equation as following:

$$\det(A - \lambda I) = 0. \quad n=2.$$

$$\det\begin{vmatrix} (-4-\lambda) & 5 \\ 1 & (-\lambda) \end{vmatrix} = 0.$$

2. We'll define own numbers of the matrix A or the roots of the characteristic equation:

$$\lambda^2 + a_1\lambda + a_2 = 0.$$

$$(4+\lambda)\lambda - 5 = \lambda^2 + 4\lambda - 5 = 0$$

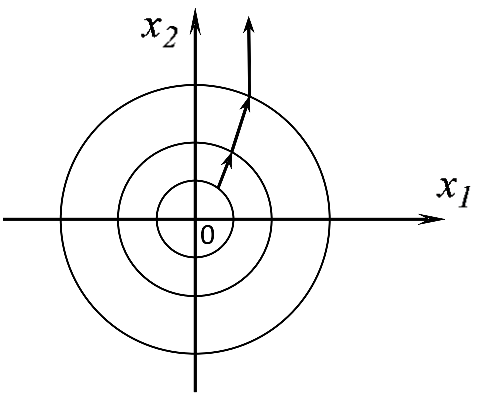
$$\lambda_1 = -5; \quad \lambda_2 = 1.$$

3. We should make of the conclusion on stability of dynamic system, using the theorems Lyapunov's.

Conclusion: this dynamic system is not steady on Lyapunov's, as a real part of the second root the characteristic equation is positive:

$$\lambda_1 = -5; \quad \lambda_2 = 1.$$

Geometrical interpretation



Task: Let's description of dynamic system is given in state space of condition in the matrix form of the following kind:

$$\dot{X} = AX + BU,$$

where matrixes *A and B* (on variant), $U(t) \equiv 0$.

Determine stability of dynamic system according to Lyapunov. Give of the geometrical interpretation.

Exercises by variants:

1)

$A = \begin{bmatrix} -2 & 5 \\ 5 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$,

2)

$A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -2 \\ +4 \end{bmatrix}$,

3)

$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$,

4)

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ +7 \end{bmatrix}$,

5)

$A = \begin{bmatrix} 1 & -1 \\ 7 & 9 \end{bmatrix}$ $B = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$,

6)

$A = \begin{bmatrix} 2 & 6 \\ 8 & 4 \end{bmatrix}$ $B = \begin{bmatrix} +1 \\ -7 \end{bmatrix}$,

7)

$A = \begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$,

8)

$A = \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$,

9)

$A = \begin{bmatrix} 10 & 11 \\ 14 & 13 \end{bmatrix}$ $B = \begin{bmatrix} -5 \\ +4 \end{bmatrix}$,

10)

$A = \begin{bmatrix} 9 & 5 \\ 3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} -4 \\ +2 \end{bmatrix}$, 11)

A = $\begin{bmatrix} 3 & 5 \\ 11 & 9 \end{bmatrix}$ B = $\begin{bmatrix} -4 \\ +6 \end{bmatrix}$,

12)

A = $\begin{bmatrix} -5 & 3 \\ 3 & -5 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 4 \end{bmatrix}$,